

Q. Solve the following minimal assignment problem

Man \rightarrow	1	2	3	4
Job \downarrow				
I	12	30	21	15
II	18	33	9	31
III	44	25	24	21
IV	25	30	28	14

Solution

Step I:

Let us find the smallest element in each row and subtract it from all the elements of the corresponding row such that each row contains at least one zero

Man \rightarrow Job \downarrow	1	2	3	4
I	0	18	9	3
II	9	24	0	22
III	23	4	3	0
IV	9	16	14	0

from the given effectiveness matrix.

For Row 1,

minimum or smallest element = 12.

\therefore subtracting 12 from each element of first row,

i.e. $12-12=0$, $30-12=18$, $21-12=9$, $15-12=3$

similarly for Row 2, 3 and 4.

Step II:

Starting from the first column find the column or columns having no zero elements and subtract the smallest element of each column to all elements of the corresponding column.

Man → Job ↓	1	2	3	4
I	0	14	9	3
II	9	20	0	22
III	23	0	3	0
IV	9	12	14	0

Step III:

Starting from the first row find the row having only one zero and mark it as □ and cross (X) all other zeros in the corresponding column.

Man → Job ↓	1	2	3	4
I	□	14	9	3
II	9	20	□	22
III	23	0	3	X
IV	9	12	14	□

Step IV:

Starting from the first column, find the column having only one unmarked zero and mark it \square and cross all

Man \ Job ↓	1	2	3	4
I	\square 0	14	9	3
II	9	20	\square 0	22
III	23	\square 0	3	1
IV	9	12	14	\square 0

Step V:

Since each row and each column have a marked zero therefore the solution is optimal and the optimal solution is

- Job I should be assigned to 1st Man
- Job II " " " " 3rd Man
- Job III " " " " 2nd Man
- Job IV " " " " 4th Man

$$\begin{aligned} \text{Min. Cost} &= 12 + 9 + 25 + 14 \\ &= 21 + 39 \\ &= \text{Rs. } 60 \end{aligned}$$

